

Dynamical and non-additive atomic van der Waals phases

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We investigate dynamical corrections to the van der Waals phase induced by the non-unitary evolution of atomic waves propagating near a perfectly conducting surface. These corrections reflect the interplay between field retardation effects and the dynamics of the external atomic degrees of freedom. The dynamical atom-surface interaction phase shift contains both local and non-local contributions. We derive the local contributions and show that they are equivalent to coarse-graining the instantaneous van der Waals potential over the time scale corresponding to the round-trip travel time of light between atom and surface. We show that the non-local phase contributions are non-additive, and propose to use this property in a multiple-path van der Waals interferometer in order to isolate them from the standard (and much larger) quasi-static van der Waals phase.

Atom interferometers [1] play a leading role in the development of precision experiments, enabling one to build extremely accurate devices for frequency metrology or for inertial sensing [2]. In the last decade, atom interferometers have been used to probe atom-surface interactions in the van der Waals (vdW) regime [3], turning atom optics into a promising field for the experimental investigation of dispersive forces [4].

The effect of surface interactions onto atomic waves propagating near a conducting plate is commonly treated by means of the vdW (or Casimir-Polder at longer distances) potential taken at the instantaneous atomic position. In this description, the external atomic waves are treated as a closed quantum system driven by conservative forces. We have shown recently [5] that such an approach is incomplete, because the external atomic degrees of freedom (d.o.f.s) are coupled to the internal dipole and electromagnetic field fluctuations. Thus, the atomic waves propagating in the vicinity of a conducting surface behave essentially as an open quantum system coupled to an environment (the dipole and electromagnetic field d.o.f.s) [6]. In addition to the expected decoherence [7], the non-unitary evolution of the atomic waves gives rise to specific atomic phase shifts. A similar real phase shift has been discussed in the context of geometrical phases in spin-boson systems coupled to an environment [8].

An open system analysis of the coupling between the moving atom and its environment indeed reveals a dynamical correction [5] in addition to the standard phase arising from the vdW potential taken at the instantaneous position. This phase correction turns out to be the sum of a local contribution, associated to individual paths in the interferometer, and a non-local double-path (DP) phase associated to pairs of paths.

In this letter, we start by providing a very simple interpretation of the local correction as a coarse-graining of the instantaneous vdW potential over the time scale corresponding to the round-trip propagation of light between atom and surface. For non-relativistic velocities, this provides a small correction that might be extremely hard to isolate from the much larger standard vdW

phase.

On the other hand, we show here that the non-local DP phase can be isolated from the standard vdW phase provided that we consider a multiple-path interferometer [9] rather than the double-arm Mach-Zehnder atom interferometer discussed in [5]. For achieving this goal, we propose to use a key property of the non-local DP phase: its non-additivity. We take the three-arm interferometer shown in Fig. 1 as the simplest example illustrating the non-additive nature of the non-local phase. In standard interferometry, one associates well defined phases to individual paths, and as a consequence we are allowed, for instance, to add phase coherences between arms 1 and 3 and 3 and 2 to find the phase coherence between arms 1 and 2. However, this is no longer the case when taking the non-local correction into account, because the DP phase coherences are associated to pairs of paths rather than to individual ones.

The three paths propagating in the half space $z > 0$ interact between $t = 0$ and $t = T$ with a nearby perfectly conducting plate located in the plane $z = 0$ as shown in Fig. 1. In addition to the dispersive interaction with the plate, atoms are driven by an external potential $V_{\text{ext}}(\mathbf{r})$, linear or quadratic in position. We assume the atomic motion non-relativistic. The atomic state is initially a sum of three Gaussian wave-packets with negligible overlap, $|\psi(0)\rangle = \frac{1}{\sqrt{3}} \sum_{k=1}^3 |\psi_k(0)\rangle$, with $\langle \mathbf{r} | \psi_k(0) \rangle = wp(\mathbf{r}, \mathbf{r}_0 k, \mathbf{p}_0 k, \mathbf{w}_0 k)$. These packets have a central position $\mathbf{r}_0 k$, momentum $\mathbf{p}_0 k$ and a width $\mathbf{w}_0 k$: $wp(\mathbf{r}, \mathbf{r}_0, \mathbf{p}_0, \mathbf{w}_0) = \prod_{\eta=x,y,z} (1/\sqrt{\pi w_{0\eta}}) e^{-(\mathbf{r}-\boldsymbol{\eta})^2/2w_{0\eta}^2 + i\mathbf{p}_0\boldsymbol{\eta}(\mathbf{r}-\boldsymbol{\eta})/\hbar}$. We consider a sufficiently dilute sample, so that atomic interactions effects can be neglected.

We first present the standard analysis of this atom interferometer by means of an instantaneous vdW potential $V_{\text{vdW}}(\mathbf{r})$. Provided that the vdW potential on the atoms is weak enough so as to make dispersion effects negligible, an excellent approximation in the experimental conditions of Ref. [3], one can apply the *ABCD* propagation method [10] for atomic waves in quadratic potentials: at

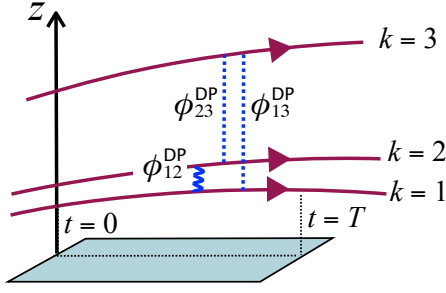


FIG. 1: (color online). Three-arm atom interferometer near a conducting plate at $z = 0$. When the arm $k = 3$ is further away from the plate, the non-local double-path phases ϕ_{23}^{DP} and ϕ_{13}^{DP} are much smaller than ϕ_{12}^{DP} thus enhancing the non-additivity effect.

any time $t > 0$, each atomic wave-packet is given by

$$|\psi_k(t)\rangle = |\chi_k(t)\rangle e^{i[\varphi_k^{(0)}(t) + \varphi_k^{(vdW)}(t)]} \quad (1)$$

with a time-dependent Gaussian $\langle \mathbf{r} | \chi_k(t) \rangle = wp(\mathbf{r}, \mathbf{r}_k(t), \mathbf{p}_k(t), \mathbf{w}_k(t))$. The precise value of the width vector $\mathbf{w}_k(t)$ is not important for the coming discussion. The average atomic position $\mathbf{r}_k(t)$ and momentum $\mathbf{p}_k(t)$ follow the classical equations of motion with the initial conditions $\mathbf{r}_k(0) = \mathbf{r}_{0k}$ and $\mathbf{p}_k(0) = \mathbf{p}_{0k}$ associated with the central trajectory corresponding to path k ($k = 1, 2, 3$). More important for our discussion are the phase contributions in Eq. (1). The phase $\varphi_k^{(0)}(t)$ collects the free propagation and external potential effects, whereas $\varphi_k^{(vdW)}(t)$ accounts for the dispersive atom-surface interaction. From now on, we focus on the phase accumulated between the instants $t = 0$ and $t = T$, omitting explicit reference to time T to alleviate notations. The phase $\varphi_k^{(0)}$ is given by the following integral along the trajectory k : $\varphi_k^{(0)} = \frac{1}{\hbar} \int_0^T dt \left(\frac{\mathbf{p}_k^2(t)}{2m} - E(t) - V_{\text{ext}}(\mathbf{r}_k(t)) \right)$, where $E(t)$ is the internal atomic energy at time t . In this standard approach, the atom-surface interaction simply yields an additional phase shift given by the integration of the vdW potential $V_{\text{vdW}}(z)$ taken at the instantaneous atomic positions along the path k :

$$\varphi_k^{(vdW)} = -\frac{1}{\hbar} \int_0^T dt V_{\text{vdW}}(z_k(t)) \quad (2)$$

The density matrix corresponding to the atomic state at time T computed within the standard *ABCD* approach is then given by

$$\rho(T) = \rho_{\text{diag}}(T) + \frac{1}{3} \left(\sum_{j < k}^3 |\chi_j(T)\rangle \langle \chi_k(T)| e^{i\phi_{jk}^{\text{st}}} + \text{H.c.} \right) \quad (3)$$

with $\rho_{\text{diag}}(T) \equiv \frac{1}{3} \sum_k |\chi_k(T)\rangle \langle \chi_k(T)|$ and H.c. representing the Hermitian conjugate. We focus here on the stan-

dard phase coherences ϕ_{jk}^{st} ,

$$\phi_{jk}^{\text{st}} = \phi_{jk}^{(0)} + \varphi_j^{(\text{vdW})} - \varphi_k^{(\text{vdW})}, \quad (4)$$

$$\phi_{jk}^{(0)} \equiv \varphi_j^{(0)} - \varphi_k^{(0)}. \quad (5)$$

They (obviously) satisfy additivity:

$$\phi_{jk}^{\text{st}} = \phi_{j\ell}^{\text{st}} + \phi_{\ell k}^{\text{st}} \quad (6)$$

for any $j, k, \ell = 1, 2, 3$, since they originate from phases associated to individual paths in Eq. (1).

We now analyse the multiple-path atom interferometer as an open quantum system, building on our recent work [5], and show that the additivity condition (6) no longer holds. We start from the full quantum system, whose dynamics is described by the Hamiltonian $\hat{H} = \hat{H}_E + \hat{H}_D + \hat{H}_F + \hat{H}_{AF}$, including the external (\hat{H}_E), internal (\hat{H}_D) and electromagnetic field (\hat{H}_F) d.o.f.s. The interaction Hamiltonian, which reads in the electric dipole approximation $\hat{H}_{AF} = -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\hat{\mathbf{r}}_a)$, couples the atomic center-of-mass $\hat{\mathbf{r}}_a$ to the internal dipole $\hat{\mathbf{d}}$ and the electric field $\hat{\mathbf{E}}$.

The external atomic waves are described by the reduced atomic density matrix obtained after coarse-graining over the field and internal atomic d.o.f.s. These play the role of an environment, whose effect on the atomic waves is captured by an influence phase $S_{\text{IF}}[\mathbf{r}_j, \mathbf{r}_k]$ [5]:

$$\rho(T) = \rho_{\text{diag}}(T) + \frac{1}{3} \left(\sum_{j < k}^3 |\chi_j(T)\rangle \langle \chi_k(T)| e^{i(\phi_{jk}^{(0)} + \frac{1}{\hbar} S_{\text{IF}}[\mathbf{r}_j, \mathbf{r}_k])} + \text{H.c.} \right) \quad (7)$$

The complex influence phase $\frac{1}{\hbar} S_{\text{IF}}[\mathbf{r}_j, \mathbf{r}_k]$, evaluated along the central atomic trajectories j and k (a valid approximation for narrow wave-packets), describes completely the atom-surface interaction effects. Its imaginary part corresponds to the plate-induced decoherence, and its real part gives the atomic phase shift arising from surface interactions. This phase contains local contributions involving a single path (SP) at a time, and a non-local double-path (DP) contribution involving simultaneously two paths:

$$\frac{1}{\hbar} \text{Re}[S_{\text{IF}}[\mathbf{r}_j, \mathbf{r}_k]] = \varphi_j^{\text{SP}} - \varphi_k^{\text{SP}} + \phi_{jk}^{\text{DP}}. \quad (8)$$

In this letter, we provide explicit analytical results for the single and double-path phase contributions in the short-distance van der Waals limit $\omega_0 z_k/c \ll 1$, which yields larger phase shifts and matches the conditions of the experiments performed so far [3]. In this regime, the dominant contribution comes from the Hadamard (or symmetric) dipole correlation function (\hat{d} is any Cartesian component of the vector operator $\hat{\mathbf{d}}$)

$$G_{\hat{d}}^H(t, t') \equiv \frac{1}{\hbar} \langle \{\hat{d}(t), \hat{d}(t')\} \rangle, \quad (9)$$

which contains the information about the quantum dipole fluctuations, whereas the Hadamard electric field correlation function yields a negligible contribution.

In the short-distance limit, the relevant field correlation function is the retarded Green's function representing the electric field linear response susceptibility to the fluctuating dipole source:

$$\mathcal{G}_{\hat{E}}^R(\mathbf{r}, t; \mathbf{r}', t') \equiv \frac{i}{\hbar} \theta(t - t') \sum_{\eta=x,y,z} \langle [\hat{E}_{\eta}(\mathbf{r}, t), \hat{E}_{\eta}(\mathbf{r}', t')] \rangle, \quad (10)$$

with $\theta(t)$ representing the Heaviside step function. $\mathcal{G}_{\hat{E}}^R(\mathbf{r}, t; \mathbf{r}', t')$ is the sum of two contributions: the free space Green's function, which represents the direct propagation from $\mathbf{r}' = (x', y', z')$ at time t' to \mathbf{r} at time t and does not contribute to the surface interaction; and the scattered Green's function $\mathcal{G}_{\hat{E}}^{R,S}$, which accounts for the propagation containing one reflection at the surface [11]. The latter is written in terms of the source point image $\mathbf{r}'_I = (x', y', -z')$, represented in Fig. 2, and vanishes outside the light cone defined by the condition $\tau \equiv t - t' = |\mathbf{r} - \mathbf{r}'_I|/c$.

We first address the local single-path phases φ_k^{SP} in (8). From the general expression for the SP phase given in [5], we find in the short-distance limit

$$\varphi_k^{\text{SP}} = \frac{1}{4} \int dt dt' \Theta_P^k(t') G_d^H(t, t') \mathcal{G}_{\hat{E}}^{R,S}(\mathbf{r}_k(t), t; \mathbf{r}_k(t'), t') \quad (11)$$

where the function $\Theta_P^k(t')$ is equal to one when the atomic position $\mathbf{r}_k(t')$ is above the plate and zero elsewhere. Note that this condition automatically bounds the integration domain for the time t , since the electric field response function $\mathcal{G}^{R,S}(\mathbf{r}_k(t), t; \mathbf{r}_k(t'), t')$ yields non-zero values only if the four-position $(\mathbf{r}_k(t), t)$ is on the light cone issued from the image four-position $(\mathbf{r}_{Ik}(t'), t')$. Eq. (11) shows that the single path vdW phase arises from the fluctuating dipole at time t' , $d(t')$, which produces an electric field propagating from its source point $\mathbf{r}' = \mathbf{r}_k(t')$ to the new atomic position $\mathbf{r} = \mathbf{r}_k(t)$ after bouncing off the plate (see Fig. 2), where it interacts with the new atomic dipole $d(t)$, still correlated to the older dipole value $d(t')$. Clearly, the dipole memory time must be larger than the time delay corresponding to the round-trip light propagation between atom and surface, $\tau = t - t' = |\mathbf{r}_k(t' + \tau) - \mathbf{r}_{Ik}(t')|/c \approx 2z_k(t')/c$, a condition easily met in the short-distance limit. Here we model the internal dipole as an harmonic oscillator in order to derive simple analytical results: $G_d^H(t, t') = \omega_0 \alpha(0) \cos[\omega_0(t - t')]$, where $\alpha(0)$ represents the zero-frequency atomic polarizability.

Combining this result with the analytic expression for the field Green's function $\mathcal{G}_{\hat{E}}^{R,S}$ [5], we derive from (11) (ϵ_0 = vacuum permittivity)

$$\varphi_k^{\text{SP}} = \frac{\omega_0 \alpha(0)}{32\pi\epsilon_0} \int_0^T dt' \frac{dt'}{\bar{z}_k^3(t')} \quad (12)$$

The distance $\bar{z}_k(t') = \frac{1}{2}(z_k(t') + z_k(t' + \tau))$ is defined in terms of the round-trip propagation time $\tau \approx 2z_k(t')/c$. We neglect second-order terms in the atomic velocity and assume that the vertical atomic acceleration is not exceedingly large ($\ddot{z}_k(t) \ll c^2/z_k(t) \simeq 5 \times 10^{24} \text{m.s}^{-2}$ for a plate distance $z_k = 20 \text{nm}$). Using the expression for the vdW potential $V_{\text{vdW}}(z) = -\hbar\omega_0\alpha(0)/(32\pi\epsilon_0 z^3)$, the SP phase (12) is then expressed directly in terms of the quasi-static standard vdW phase (2) plus a dynamical first-order correction proportional to the potential gradient $V'_{\text{vdW}}(z_k(t))$:

$$\varphi_k^{\text{SP}} = \varphi_k^{(\text{vdW})} - \frac{1}{\hbar} \int_0^T dt V'_{\text{vdW}}(z_k(t)) \frac{z_k(t) \dot{z}_k(t)}{c} \quad (13)$$

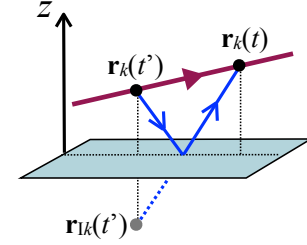


FIG. 2: (color online). Effect of retardation on the van der Waals phase: the vdW potential as seen by the atomic wavepacket is coarse-grained over the time interval corresponding to the round-trip propagation of light between atom and surface, producing a phase correction proportional to the vertical atomic velocity.

We may also cast the expression (13) for φ_k^{SP} in the same form of (2), provided that we replace the potential taken at the instantaneous position $V_{\text{vdW}}(z_k(t))$ by its coarse-grained average over the round-trip time $\tau = 2z_k(t)/c$: $\bar{V}_{\text{vdW}}(z_k(t)) = \frac{1}{\tau} \int_t^{t+\tau} dt' V_{\text{vdW}}(z_k(t'))$. In other words, the position at which the vdW potential is taken cannot be resolved below the scale $\Delta z = 2\dot{z}_k(t)z_k(t)/c$ representing the atomic vertical displacement during the round-trip time $2z_k(t)/c$, as illustrated by Fig. 2. As far as local phases are concerned, the averaging of the vdW potential thus captures the interplay between field retardation and external atomic motion.

The resulting dynamical correction to the SP phase, given by the second term in the r.-h.-s. of (13), turns out to depend on the endpoints only. For the typical non-relativistic velocities employed in atom interferometers, it is a very small phase, smaller than the standard quasi-static vdW phase (2) by a factor of the order of \dot{z}_k/c [12]. It thus seems to be out of experimental reach, for it would be extremely difficult to isolate this phase contribution from the much larger standard vdW phase.

The non-local double-path phase contributions ϕ_{jk}^{DP} in (8) are also of first-order in \dot{z}_k/c and thus much smaller than the standard vdW phase. However, in contrast to the SP phases discussed above, they are non-additive as

shown in the following, which could be used to isolate them from the main contribution.

The physical origin of ϕ_{jk}^{DP} is similar to the local phase dynamical correction discussed in connection with (11), except that it involves propagation between two different wave-packets. More precisely, ϕ_{jk}^{DP} is derived as the difference between the propagation integrals analogous to (11) connecting wave-packet k to j and j to k [5]:

$$\phi_{jk}^{\text{DP}} = \frac{1}{4} \int_0^T dt \int_0^T dt' G_d^H(t, t') \left[\mathcal{G}_{\tilde{E}}^{R,S}(\mathbf{r}_j(t), t; \mathbf{r}_k(t'), t') - \mathcal{G}_{\tilde{E}}^{R,S}(\mathbf{r}_k(t), t; \mathbf{r}_j(t'), t') \right]. \quad (14)$$

ϕ_{jk}^{DP} vanishes in the quasi-static limit, since the two propagation integrals in (14) cancel each other exactly to zeroth-order of \dot{z}_k/c . In contrast with the SP phase (13), ϕ_{jk}^{DP} is thus a pure dynamical phase shift, arising from the asymmetry between the propagations from wave-packet j to k and vice-versa, which is brought into play by the finite speed of light and the vertical motions of each packet.

In order to derive an explicit analytical result from (14), we assume that the different atomic paths are in the same vertical plane and share the same velocity component parallel to the plate. On the other hand, we take arbitrary non-relativistic motions along the perpendicular direction, which correspond to the functions $z_k(t)$, under the short-distance condition $\omega_0 z_k/c \ll 1$. Neglecting as before terms of order $(\dot{z}_k/c)^2$, we derive from (14)

$$\phi_{jk}^{\text{DP}} = 3 \frac{\omega_0 \alpha(0)}{4\pi\epsilon_0 c} \int_0^T dt \frac{\dot{z}_k(t) - \dot{z}_j(t)}{(z_j(t) + z_k(t))^3} \quad (15)$$

Note that this phase is independent of the velocity component parallel to the conductor plane. This follows from translational invariance parallel to the plate and from the condition of perfect conductivity. Because it depends linearly on the speed of each trajectory, ϕ_{jk}^{DP} is invariant under time dilatation $z_j \rightarrow \tilde{z}_j(t) \equiv z_j(\Lambda t)$, $j = 1, 2$, $T \rightarrow T/\Lambda$, with Λ arbitrary.

We now stress the main point of this letter: the double-path phase ϕ_{jk}^{DP} as given by (15) is non-additive, since the denominator in its r.h.s. does not allow one to isolate separate contributions from paths j and k , a signature of the non-local nature of ϕ_{jk}^{DP} . This non-additivity is enhanced when considering a geometry for which the third path is much further away from the plate than the first and second paths (see Fig. 1): we take $z_3(t) \gg z_1(t), z_2(t)$ and assume that the differences in vertical atomic velocities are of the same order of magnitude $\dot{z}_1(t) - \dot{z}_2(t) \sim \dot{z}_2(t) - \dot{z}_3(t)$. It then follows from (15) that $\phi_{13}^{\text{DP}} + \phi_{32}^{\text{DP}} \ll \phi_{12}^{\text{DP}}$: the non-additivity is maximal in this case.

One can actually use the non-additivity in order to isolate the non-local dynamical corrections from the other phase contributions. For the three-arm interferometer shown in Fig. 1, we propose to measure separately the three independent phase coherences appearing in Eq. (7), $\phi_{jk} \equiv \phi_{jk}^{(0)} + \frac{1}{\hbar} \text{Re}[S_{\text{IF}}[\mathbf{r}_j, \mathbf{r}_k]]$ with $j, k = 1, 2, 3$, $j \neq k$, by performing interferometric measurements between the different pairs of arms. Using (8), we find that the (maximal) violation of phase additivity gives the desired non-local double-path shift ϕ_{12}^{DP} :

$$\phi_{12}^{\text{DP}} \approx \phi_{12} - (\phi_{13} + \phi_{32}). \quad (16)$$

This approach removes all the additive phases, leaving only the non-local dynamical correction to the vdW phase. We have studied [5] its amplitude for ^{87}Rb atoms close to a metallic plate, taking path 1 to be parallel to the plate at a constant distance $z_1 = 20$ nm, similar to the experimental value reported in Ref. [3]. In this configuration, the integrated double-path phase (15) depends only on the end-point positions of path 2. We take $z_2(0) = z_1$ and $z_2(T) \gg z_1$ to find $\phi_{12}^{\text{DP}} \simeq 3 \times 10^{-7}$ rad. This is beyond the state of the art in atom interferometry, but still bigger than systematics considered in the best atom interferometers [2].

To conclude, we have used an open system theory of atom interferometers to derive dynamical corrections to the standard van der Waals phase shifts. We have shown that the interplay between field retardation effects and the external atomic dynamics generates first-order dynamical corrections. The local corrections, associated to individual paths, turn out to be equivalent to coarse-graining the vdW potential over a time scale corresponding to the round-trip travel time of the atom-surface interaction. The non-local phase corrections are associated to pairs of interferometer paths rather than to individual ones, and are of the same order of magnitude of the local corrections. More importantly, they are generally non-additive, a distinctive characteristic associated to non-locality. We have proposed a method to isolate them from other phase shifts in a three-path atom interferometer. These results show that coupling with the environment may induce, in addition to decoherence, phase shifts with unusual properties in atom optics.

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[1] A. D. Cronin, J. Schmiedmayer and D. E. Pritchard, *Rev. Modern Phys.* **81**, 1051 (2009) and references therein.

[2] J. M. Hogan, D. M. S. Johnson, M. A. Kasevich, in *Proc. Int. School of Physics Enrico Fermi* (2007) and references

- therein.
- [3] J. D. Perreault and A. D. Cronin, Phys. Rev. Lett. **95**, 133201 (2005); S. Lepoutre, H. Jelassi, V. P. A. Lonij, G. Trénec, M. Büchner, A. D. Cronin and J. Vigué, Europhys. Lett. **88**, 20002 (2009).
 - [4] P. Wolf, P. Lemonde, A. Lambrecht, S. Bize, A. Landragin, and A. Clairon, Phys. Rev. A **75**, 063608 (2007); S. Pelisson, R. Messina, M.-C. Angonin, and P. Wolf, Phys. Rev. A **86**, 013614 (2012).
 - [5] F. Impens, R. O. Behunin, C. Ccapa Ttira, and P. A. Maia Neto, Europhysics Lett. **101**, 60006 (2013).
 - [6] R. O. Behunin, and B.-L. Hu, J. Phys. A: Math. Theor. **43**, 012001 (2010); Phys. Rev. A **82**, 022507 (2010); Phys. Rev. A **84**, 012902 (2011).
 - [7] A. Stern, Y. Aharonov, and Y. Imry, Phys. Rev. A **41**, 3436 (1990); J. R. Anglin and W. H. Zurek, in *Dark Matter in Cosmology, Quantum Measurements, Experimental Gravitation*, pp. 263-270, edited by R. Ansari, Y. Giraud-Heraud and J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, 1996); F. D. Mazzitelli, J. P. Paz and A. Villanueva, Phys. Rev. A **68**, 062106 (2003); P. Sontentag and F. Hasselbach, Phys. Rev. Lett. **98**, 200402 (2007); S. Scheel and S. Y. Buhmann, Phys. Rev. A **85**, 030101(R) (2012).
 - [8] R. S. Whitney, Y. Makhlin, A. Shnirman and Y. Gefen, Phys. Rev. Lett. **94**, 070407 (2005); F. C. Lombardo and P. I. Villar, Phys. Rev. A **74**, 042311 (2006).
 - [9] M. Weitz, T. Heupel, and T. W. Hänsch, Phys. Rev. Lett. **77**, 2356 (1996); H. Hinderthür et al., Phys. Rev. A **56**, 2085 (1997); H. Hinderthür et al., Phys. Rev. A **59**, 2216 (1999); F. Impens, C. J. Bordé, Phys. Rev. A **80** 031602 (2009); M. Robert-de-Saint-Vincent et al., Europhysics Lett. **89** 10002 (2010); F. Impens, F. Pereira dos Santos, and C. J. Bordé, New J. Phys. **13**, 065024 (2011).
 - [10] C. J. Bordé, Metrologia **39**, 435 (2002).
 - [11] J. M. Wylie and J. E. Sipe, Phys. Rev. A **30**, 1185 (1984); ibid. **32**, 2030 (1985).
 - [12] In addition, our dynamical phase correction is also smaller than the correction due to the internal atomic dynamics, which corresponds to the time scale $1/\omega_0$. The ratio between the external center-of-mass (discussed here) and internal corrections is of the order of the inverse ratio between the corresponding time scales: $(\dot{z}_k/c)(\lambda_0/z_k) \ll 1$ ($\lambda_0 = 2\pi c/\omega_0$ represents the typical atomic transition wavelength), which is typically very small since z_k should be much larger than the atomic length scale in our formalism for dispersive interactions.